

## **Durability Limit States of Concrete Structures: Probabilistic Modeling**

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### **ABSTRACT**

In the context of performance-based approaches, sustainability and whole life costing, the concrete structures durability issue has recently gained considerable attention. In the present paper a list and explanation of durability limit states (DLS) specialized for concrete structures and a combination of initiation and propagation durability states are introduced. The assessment of such limit states is based on degradation modeling and a probabilistic approach, enabling the assessment of service life and the relevant reliability level.

In this paper the durability limit states specialized for concrete structures are discussed and the review of some available analytical models for reinforced concrete degradation assessment, their randomization and the use of simulation techniques are provided. Some numerical examples using a special software tool are presented.

### **KEYWORDS**

Durability limit states, probabilistic modeling, carbonation, chloride ingress, corrosion

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## 1 INTRODUCTION

During the last twenty years the concrete structures service life issue has been given considerable attention. This is clearly reflected also in standardization activities: the ISO/WD 13823 and fib-Model Code [2006] which are based on probabilistic approaches and will introduce the design of structures for durability – i.e. a time-dependent limit state approach with service life consideration.

## 2 DURABILITY LIMIT STATES FOR REINFORCED CONCRETE

When considering the degradation of reinforced concrete structures, the corrosion of reinforcement is the dominant effect. Usually the initiation and propagation periods are assessed. The former is the time from concrete casting to the moment when the reinforcement is no longer passivated. The latter includes the period since corrosion initiation. Generally, the limit state condition may be written as:

$$P_f(t_D) = P\{B(t_D) - A(t_D) \leq 0\} \leq P_d \quad (1)$$

where  $A$  is the action effect,  $B$  is the barrier at time  $t_D =$  design service life and  $P_d$  is the design (acceptable, target) probability value. More about the limit states for durability (DLS) see e.g. in Teplý et al. [2008] or in fib-Model Code [2006]. Let us list possible actions and barriers usable for DLS:

Carbonation:  $B = a$  is a concrete cover and  $A = x_c$  is the depth of carbonation at time  $t_D$ .

Chloride ingress:  $B = C_{cr}$  is a critical concentration of dissolved Cl<sup>-</sup> leading to steel depassivation and  $A = C_a$  is a concentration of Cl<sup>-</sup> at the reinforcement at time  $t_D$ .

Concrete cracking due to corrosion:  $B = \sigma_{cr}$  is a critical tensile stress that initiates a crack in concrete (on the interface with a reinforcing bar) and  $A = \sigma$  is a tensile stress in concrete or  $B = w_{cr}$  is a critical crack width on the concrete surface and  $A = w_a$  is a crack width on the concrete surface generated by reinforcement corrosion.

A decrease of the effective reinforcement cross-section due to corrosion:  $B = A_t$  is the reinforcement cross-sectional area at time  $t_D$  and  $A = A_{min}$  is the minimum acceptable reinforcement cross-sectional area with regard to either serviceability limit state (SLS) or ultimate limit state (ULS).

As stated in the basic design code EN 1990 [2002], the recommended value of the reliability index for SLS (irreversible state) is 1.5, which is relevant to a 50-year design service life. It should be noted that the values of  $0.8 < \beta_d \leq 1.6$  for the DLS are currently under discussion (e.g. the recommendation of the fib Model Code reads  $\beta_d = 1.3$ ). The level of reliability in the context of durability should be left to the client's decision together with the target service life - as indicated in both future documents ISO/WD 13823 and fib-Model Code.

## 3 ANALYTICAL MODELS

Several models were selected on the basis of the authors' literature survey and are briefly mentioned within the next paragraphs. Most of the models were primarily published as deterministic ones; for our purposes all of them were randomized and included in the probabilistic software FReET [Novák et al. 2003], creating a special degradation module FReET-D [Teplý et al. 2007] – see section 4.1.

### 3.1 Initiation period

The mathematical models of processes relevant for the initiation period concern the concrete carbonation and chloride ions ingress.

(i) Carbonation models introduced herein are based on the diffusion of CO<sub>2</sub> in the pore system of concrete and have been discussed in [Teplý et al. 2005, 2008]. Both, the carbonation of concretes from Portland Cement (OPC) and from blended cements (where the supplementary cementing materials – SCMs – are used) have been incorporated; more details about the latter case see in [Chromá et al. 2007], including the discussion of *k*-value concept. Model from *fib* Model Code [2006] is also included.

(ii) Chloride ingress models are also encompassed in FReET-D software module starting with the models utilizing the widely used Crank's solution to Fick's 2nd law for diffusion of chlorides through concrete, taking into account the fact that the transport of chlorides in concrete is mainly diffusion controlled, and the convection zone is relatively small [Hunkeler 2005].

The simplest models usually involve a diffusion coefficient *D* of chloride through concrete as a time and space independent input parameter. It is according to Tang & Gulikers [2007] too conservative due to the fact that *D* is naturally decreasing in time resulting from chloride binding. Therefore the value of *D* may be calculated e.g. on the basis of the formula proposed by Thomas & Bamforth [1999]. They have derived a formula for the diffusion coefficient as a function of time for (a) a control mix made of OPC only, (b) concretes with 30 % of fly ash and (c) with 70 % of slag as a partial replacement for OPC. Unfortunately, this mathematical simplification where parameter *D* is simply substituted by time-dependent *D*(*t*) without adequate clarifying the mathematical basis of diffusion may not be quite correct [Tang & Gulikers 2007]. In such case the apparent diffusion coefficient is underestimated and it will cause errors in durability design and redesign of reinforced concrete structures. Also the mathematical expression adopted by *fib* Model Code [2006] does not fulfill the correct differential equation. The proper mathematical derivation of time-dependent diffusion coefficient was published by Nilsson & Carcasses [2004] and comparison of possible errors caused by some oversimplified mathematical expressions were presented by Tang & Gulikers [2007].

Although those models based on simple Fick's 2<sup>nd</sup> law are used by engineers in practical applications due to their relatively simple mathematical expressions, however models based on the actual physical or chemical processes would be more adequate. They are often used only for research purposes owing to necessities of more sophisticated calculations and only some models were simplified into a more engineer-friendly form [Papadakis et al. 1996, Tang 2007].

### **3.2 Propagation period**

The key factor for the modeling of steel corrosion as a time dependent process is the corrosion rate which is usually expressed through current density,  $i_{corr}$ . This variable is strongly affected by ambient conditions such as humidity and temperature, moisture and oxygen availability at steel level, the degree of concrete carbonation and amount of chlorides; thus depending on the diffusion characteristics of concrete. The presented models are sorted into three basic groups (i) - (iii) according to their output information and are also implemented in FReET-D.

(i) The loss of reinforcement in time may be expressed by formulas proposed by Andrade et al. [1996] and Gonzales et al. [1995] for uniform and pitting type of corrosion, respectively. Those formulas are based on current density and parameter  $R_{corr}$  that expresses the type of corrosion. The residual reinforcement cross section in the case of pits can be predicted by the simplification into a hemispherical form as proposed by Val & Melchers [1998].

(ii) Time to cracks initiation in concrete: Complex models that describe the entire process of stress and crack development in concrete due to corrosion are rather rare; more extensively published are models predicting only the time of cracking initiation in concrete. The model for calculation of the time to crack initiation in concrete cover proposed by Morinaga [1988] is an empirical equation based on experimental data. This model takes into account only the initial bar diameter, concrete cover

thickness and rate of corrosion. It does not include any mechanical parameters of concrete. Bažant [1979] has proposed a physical-mathematical model for calculation of initiation time to corrosion cracking taking into account concrete cover depth, reinforcing bar diameter and spacing, mechanical properties of concrete, etc. Concrete is considered to be a homogeneous elastic material shaped as a thick-wall cylinder. This model also solves the time of steel depassivation due to critical chloride concentration. Bažant's model [1979] was extended by Liu & Weyers [1998] who proposed a model for time of crack initiation based on a comparison of the minimum stress required to cause cracking (which equals to the tensile strength of concrete cover) with the expansive pressure in concrete developed over time due to the growth of corrosion products. Another improvement was proposed by Maaddawy & Soudki [2007]. The authors have developed a relationship between steel mass loss and the internal radial pressure caused by rust growth. The time to corrosion cracking is estimated on the basis of Faraday's law.

(iii) Stresses and cracks development in concrete: Bhargava et al. [2005] have developed a model describing the entire cracking process after crack initiation. Five modifications of the model are proposed to reproduce various experimental trends. Li et al. [2006] have also proposed a model describing the whole cracking process in concrete due to corrosion with the possibility of calculating the width of a crack on a concrete surface.

#### **4 NUMERICAL EXAMPLE**

In the following some of the above mentioned models are used to illustrate possible practical applications. Firstly, a software tool developed by the authors for the probabilistic assessment of different DLSs is introduced.

##### **4.1 Software Tool**

The multipurpose probabilistic software FReET, for general statistical, sensitivity and reliability analysis of engineering problems, has been used for randomization of the analytical models [Novák et al. 2003]. It enables the feasible and user friendly utilization of stochastic approaches (a combination of analytical models and simulation techniques) to form the specialized software FReET-D for assessing the potential degradation of newly designed as well as existing concrete structures [Teplý et al. 2007]. Implemented degradation models may serve directly for the durability assessment of structures in the form of a simple limit state (Eq. 1), i.e. the assessment of service life and the level of relevant reliability measure. The user may create different limit conditions. For the statistical analysis of the following examples the Latin Hypercube Sampling method was applied. For the output quantities the best fit of probabilistic distribution function (PDF) was found using the Kolmogorov Smirnov goodness-of-fit test.

##### **4.2 Loss of Reinforcement Due to Corrosion Initiated by Chloride Ingress**

Let us assume the critical loss of the reinforcement area to be 10 % (such a loss may e.g. lead to the exceeding of the reliability level for the ULS or SLS – depending on the structure and loading configuration). For the calculation of steel loss in time a deterministic model proposed by Andrade et al. [1996] was applied. Time to corrosion initiation  $t_i$  was calculated on the basis of model proposed in [Papadakis et al. 1996]:  $t_i = LN(47.4; 11.5)$  years. The full description of all input parameters is given in Tables 1 and 2. The decrease in rebar diameter over time is plotted in Fig. 4.2.1. The mean values of  $t_i$  and  $t_{d,crit}$  = the time of a critical drop in rebar diameter are marked in this figure.

**Table 1.** Input parameters for the calculation of time to reinforcement depassivation.

<i>Reflection of</i>	<i>Variable</i>	<i>Unit</i>	<i>Mean value</i>	<i>COV</i>	<i>PDF</i>
Models	Uncertainty factor of model	-	1	0.15	Lognormal (2par)
Environment	CO <sub>2</sub> content in the atmosphere	mg/m <sup>3</sup>	820	0.12	Normal
	Relative humidity	%	70	0.07	Beta (a = 0, b = 100)
Concrete mix	Unit content of OPC cement	kg/m <sup>3</sup>	313	0.03	Normal
	Unit content of water	kg/m <sup>3</sup>	185	0.03	Normal
	Unit content of aggregate (0-4 mm)	kg/m <sup>3</sup>	847	0.03	Normal
	Unit content of aggregate (4-8 mm)	kg/m <sup>3</sup>	386	0.03	Normal
	Unit content of aggregate (8-16 mm)	kg/m <sup>3</sup>	625	0.03	Normal
	Specific gravity of cement	kg/m <sup>3</sup>	3100	0.02	Normal
	Specific gravity of aggregate (0-4 mm)	kg/m <sup>3</sup>	2590	0.02	Normal
	Specific gravity of aggregate (4-8 mm)	kg/m <sup>3</sup>	2540	0.02	Normal
	Specific gravity of aggregate (8-16 mm)	kg/m <sup>3</sup>	2660	0.02	Normal
Other	Concrete cover	mm	25 - 75	-	Deterministic
	Concentration of Cl <sup>-</sup> on nearest concrete surface	mol/m <sup>3</sup>	50	-	Deterministic
	Saturation concentration of Cl <sup>-</sup> in solid phase	mol/m <sup>3</sup>	140	-	Deterministic
	Threshold concentration of Cl <sup>-</sup> in liquid phase	mol/m <sup>3</sup>	13.4	-	Deterministic
	Diffusion coefficient of Cl <sup>-</sup> in infinite solution	m <sup>2</sup> /s	1.6 · 10 <sup>-9</sup>	-	Deterministic

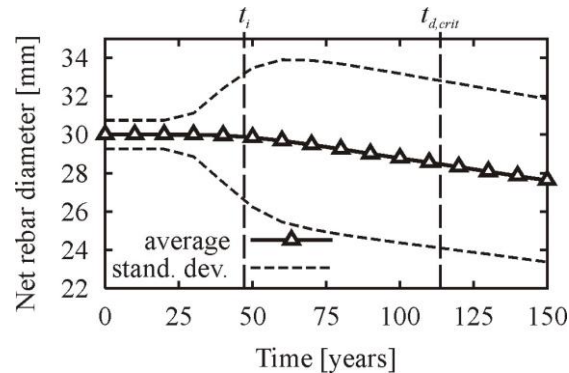
**Table 2.** Input parameters for calculation of loss of reinforcement due to corrosion.

<i>Input parameter</i>	<i>Unit</i>	<i>Mean value</i>	<i>COV</i>	<i>PDF</i>
Initial bar diameter	mm	30	0.025	Lognormal (2 par)
Time to corrosion initiation	years	47.4	0.244	Lognormal (2 par)
Current density	μA/cm <sup>2</sup>	1	0.2	Normal
Coefficient of corrosion type	-	2	-	Deterministic
Uncertainty factor of model	-	1	0.15	Lognormal (2 par)

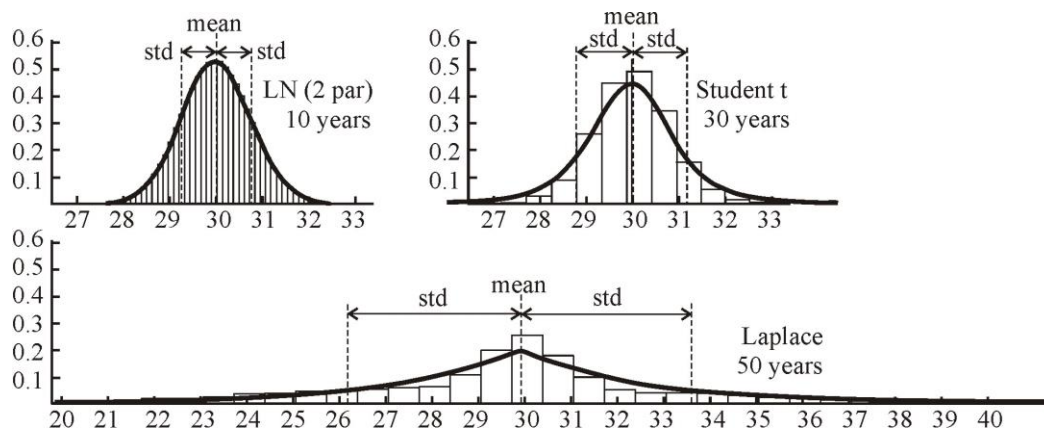
The best-fitted PDFs for output net rebar diameters are LN (2 par) for 0, 10 and 20 years, Student t for 30 years, Laplace for 40 and 50 years, LN (2 par) for 60, 70, 80, 90 and 100 years, LN (3 par) for 110 and 120 years and LN (2 par) for 130, 140 and 150 years. Chosen histograms of output parameter together with fitted PDFs are depicted in Fig. 4.2.2 showing the complexity of a statistical description of the problem solved. Note that lognormal PDFs appear for time intervals of 0-20 and 60-150 years. In the first interval the steel is not yet depassivated, while in the second time interval the steel is

already depassivated in the majority of stochastic realizations. Therefore, the standard deviation (std) of output net rebar diameter in the time interval of 0 to 20 years is given by std of the input initial bar diameter while the std in the time interval of 60-150 years is affected by all input variables and is much greater. In the intermediate part (i.e. 40-100 years) the standard deviation gradually increases (see Fig. 4.2.1).

#### 4.2.1 Net rebar diameter ( $\pm$ standard deviation) vs. time.



#### 4.2.2 Histograms of output net rebar diameter plotted in Fig. 4.2.1 (decreased due to corrosion) together with the best-fitted PDFs in chosen time steps.



## 5 FINAL REMARKS

The probabilistic durability design approach is discussed in this paper, focused on the initiation and propagation period. Appropriate limit states are explained and suitable models are briefly described. Finally, a numerical example is shown.

Concurrently, the presented approach lacks certain considerations, e.g.: (i) the spatial randomness of material and/or environmental characteristics are not considered (random fields could be a remedy), (ii) the spatial distribution of deterioration is not distinguished while assessing limit states, and (iii) combination with mechanical stress is not taken into account. The authors' ongoing research is focused in this direction partially; the effect of chloride concentration on reinforcement corrosion is also currently being studied utilizing the cellular automata technique.

## **ACKNOWLEDGMENTS**

This outcome has been achieved with the financial support of the Czech Ministry of Education, project No. 1M0579, within the activities of the CIDEAS research centre.

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